Деректер ғалымдарына арналған практикалық статистика (Python)

7-тарау. Бақылаусыз оқыту

Import required Python packages.

In [1]:

```
import math
from pathlib import Path
import pandas as pd
import numpy as np

from sklearn import preprocessing
from sklearn.decomposition import PCA
from sklearn.cluster import KMeans
from sklearn.mixture import GaussianMixture

from scipy.cluster.hierarchy import linkage, dendrogram, fcluster
from scipy.stats import multivariate_normal

import prince

import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib.colors import from_levels_and_colors
import seaborn as sns

%matplotlib inline
```

In [2]:

```
try:
    import common
    DATA = common.dataDirectory()
except ImportError:
    DATA = Path().resolve() / 'data'
```

Define paths to data sets. If you don't keep your data in the same directory as the code, adapt the path names.

In [3]:

```
SP500_DATA_CSV = DATA / 'sp500_data.csv.gz'
SP500_SECTORS_CSV = DATA / 'sp500_sectors.csv'
LOAN_DATA_CSV = DATA / 'loan_data.csv.gz'
HOUSE_TASKS_CSV = DATA / 'housetasks.csv'
```

Principal Components Analysis

A simple example

In [4]:

```
sp500_px = pd.read_csv(SP500_DATA_CSV, index_col=0)
oil_px = sp500_px[['XOM', 'CVX']]
print(oil_px.head())
```

```
                 XOM       CVX
1993-01-29 -0.016991  0.072921
1993-02-01  0.016991  0.102089
1993-02-02  0.084954  0.029168
1993-02-03  0.067964  0.058337
1993-02-04  0.034378  0.044272
```

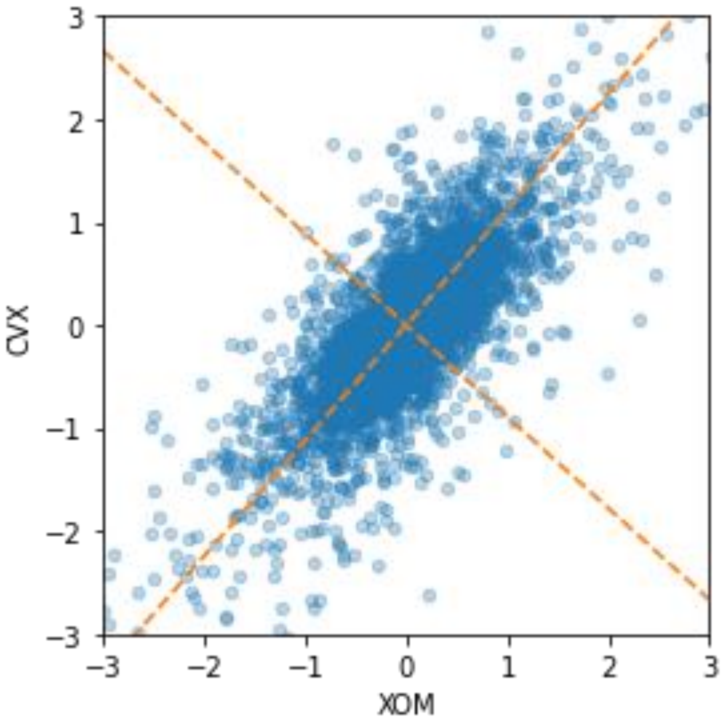
In [5]:

```
pcs = PCA(n_components=2)
pcs.fit(oil_px)
loadings = pd.DataFrame(pcs.components_, columns=oil_px.columns)
print(loadings)
```

```
        XOM       CVX
0 -0.664711 -0.747101
1  0.747101 -0.664711
```

In [6]:

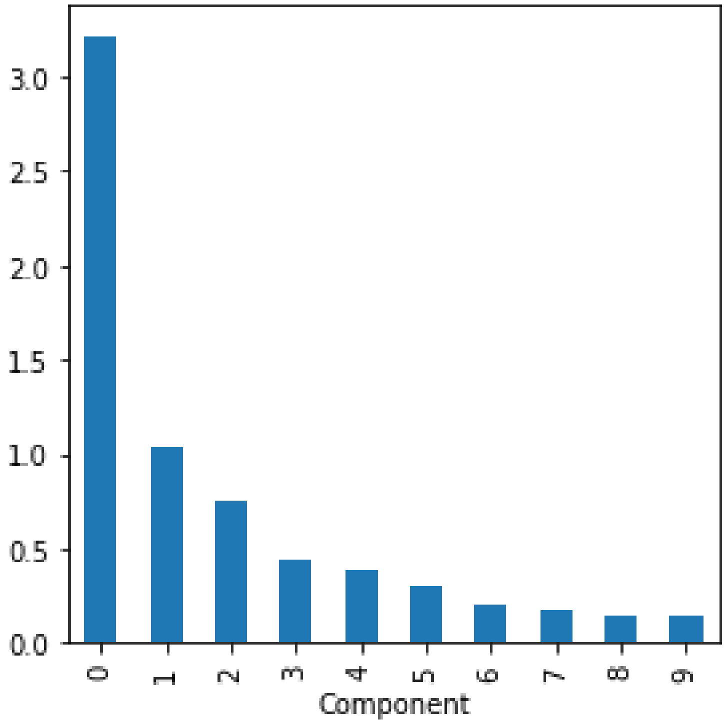
```
def abline(slope, intercept, ax):
    """Calculate coordinates of a line based on slope and intercept"""
    x_vals = np.array(ax.get_xlim())
    return (x_vals, intercept + slope * x_vals)

ax = oil_px.plot.scatter(x='XOM', y='CVX', alpha=0.3, figsize=(4, 4))
ax.set_xlim(-3, 3)
ax.set_ylim(-3, 3)
ax.plot(*abline(loadings.loc[0, 'CVX'] / loadings.loc[0, 'XOM'], 0, ax),
        '--', color='C1')
ax.plot(*abline(loadings.loc[1, 'CVX'] / loadings.loc[1, 'XOM'], 0, ax),
        '--', color='C1')

plt.tight_layout()
plt.show()
```



Interpreting principal components

In [7]:

```
syms = sorted(['AAPL', 'MSFT', 'CSCO', 'INTC', 'CVX', 'XOM', 'SLB', 'COP',
        'JPM', 'WFC', 'USB', 'AXP', 'WMT', 'TGT', 'HD', 'COST'])
top_sp = sp500_px.loc[sp500_px.index >= '2011-01-01', syms]

sp_pca = PCA()
sp_pca.fit(top_sp)

explained_variance = pd.DataFrame(sp_pca.explained_variance_)
ax = explained_variance.head(10).plot.bar(legend=False, figsize=(4, 4))
ax.set_xlabel('Component')

plt.tight_layout()
plt.show()
```



In [8]:

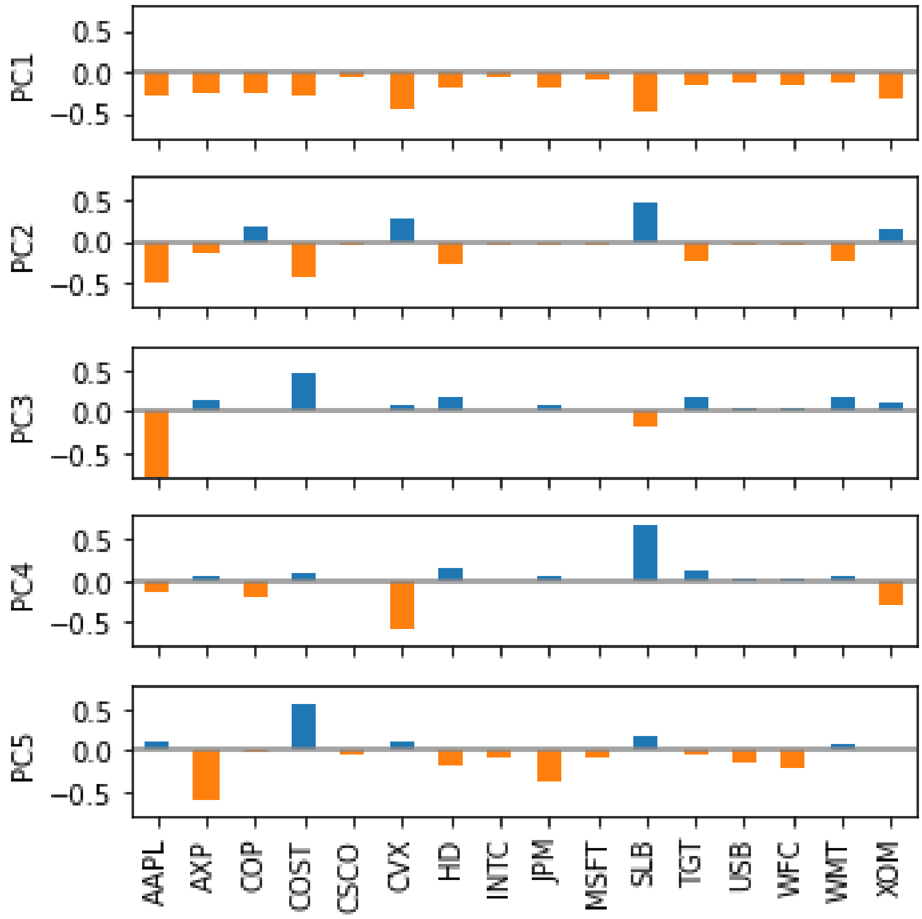
```
loadings = pd.DataFrame(sp_pca.components_[0:5, :],
                        columns=top_sp.columns)
print(loadings)
```

```
       AAPL       AXP       COP      COST      CSCO       CVX        HD  \
0 -0.300825 -0.246332 -0.261529 -0.273634 -0.064059 -0.444490 -0.207983
1 -0.505116 -0.139426  0.174212 -0.416307 -0.031939  0.289373 -0.278002
2 -0.786730  0.135458 -0.002367  0.465862 -0.007524  0.082374  0.166320
3 -0.120586  0.061814 -0.206026  0.092596  0.003904 -0.577665  0.162814
4  0.111576 -0.596666 -0.005813  0.555529 -0.039860  0.109016 -0.185488

       INTC       JPM      MSFT       SLB       TGT       USB       WFC  \
0 -0.076956 -0.196397 -0.105012 -0.481786 -0.148833 -0.116421 -0.145684
1 -0.033898 -0.040723 -0.053954  0.472494 -0.228123 -0.054796 -0.047427
2 -0.003518  0.062261  0.016248 -0.194822  0.160833  0.048976  0.041932
3 -0.001605  0.057687 -0.012558  0.680914  0.109895  0.016752  0.018614
4 -0.072047 -0.385160 -0.077135  0.181332 -0.055557 -0.155440 -0.216425
```

```
        WMT       XOM
0 -0.122304 -0.317952
1 -0.222889  0.154192
2  0.175806  0.090167
3  0.058439 -0.295204
4  0.091541  0.013277
```

In [9]:

```
maxPC = 1.01 * loadings.loc[0:5, :].abs().to_numpy().max()

f, axes = plt.subplots(5, 1, figsize=(5, 5), sharex=True)

for i, ax in enumerate(axes):
    pc_loadings = loadings.loc[i, :]
    colors = ['C0' if l > 0 else 'C1' for l in pc_loadings]
    ax.axhline(color='#888888')
    pc_loadings.plot.bar(ax=ax, color=colors)
    ax.set_ylabel(f'PC{i+1}')
    ax.set_ylim(-maxPC, maxPC)

plt.tight_layout()
plt.show()
```

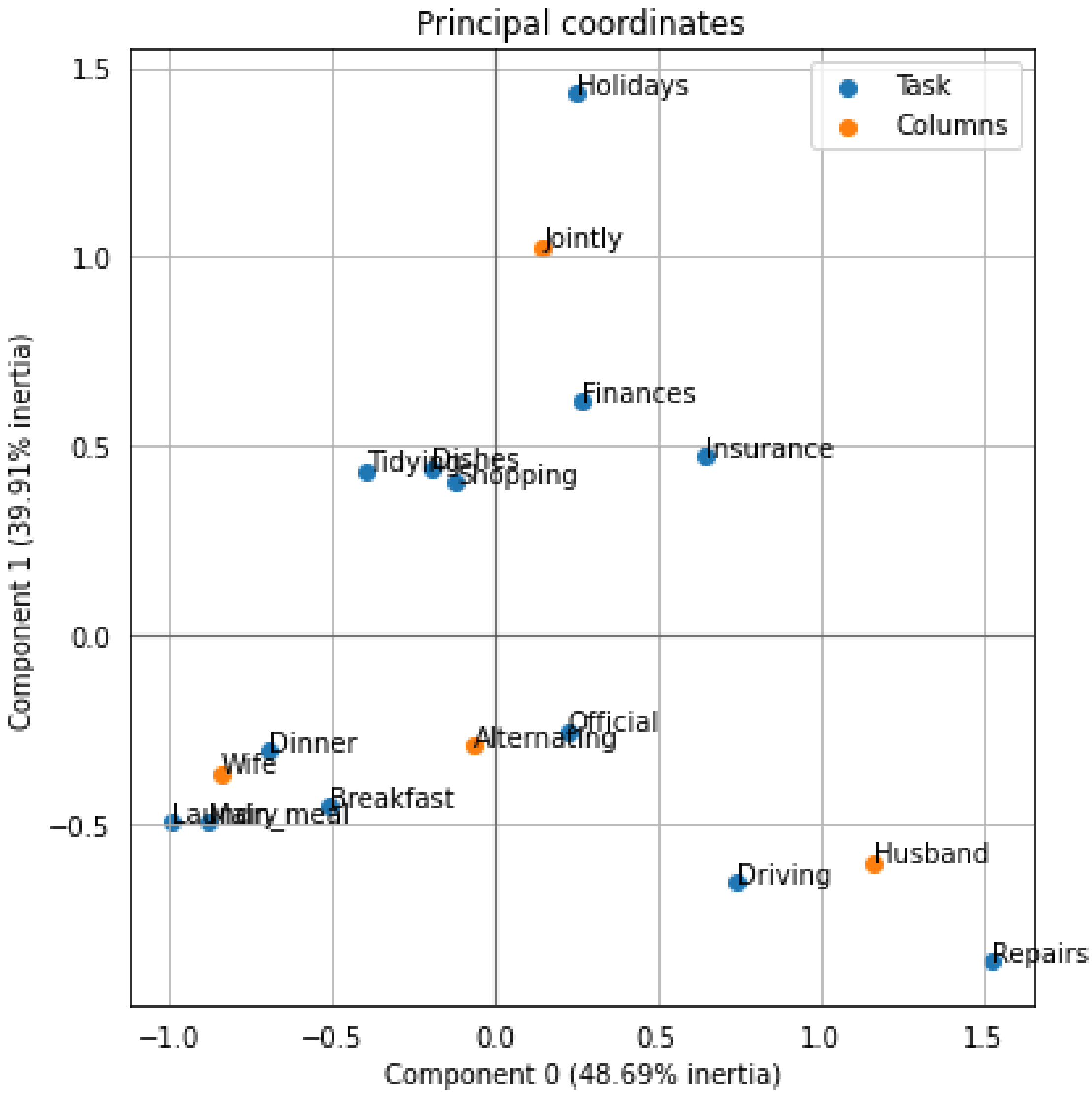


Correspondence analysis

In [10]:

```
housetasks = pd.read_csv(HOUSE_TASKS_CSV, index_col=0)

ca = prince.CA(n_components=2)
ca = ca.fit(housetasks)
```

```
ca.plot_coordinates(housetasks, figsize=(6, 6))
plt.tight_layout()
plt.show()
```



K-Means Clustering

A Simple Example

In [11]:

```
df = sp500_px.loc[sp500_px.index >= '2011-01-01', ['XOM', 'CVX']]
kmeans = KMeans(n_clusters=4).fit(df)
df['cluster'] = kmeans.labels_
print(df.head())
```

```
                 XOM       CVX  cluster
2011-01-03  0.736805  0.240681        3
2011-01-04  0.168668 -0.584516        0
2011-01-05  0.026631  0.446985        3
2011-01-06  0.248558 -0.919751        0
2011-01-07  0.337329  0.180511        3
```

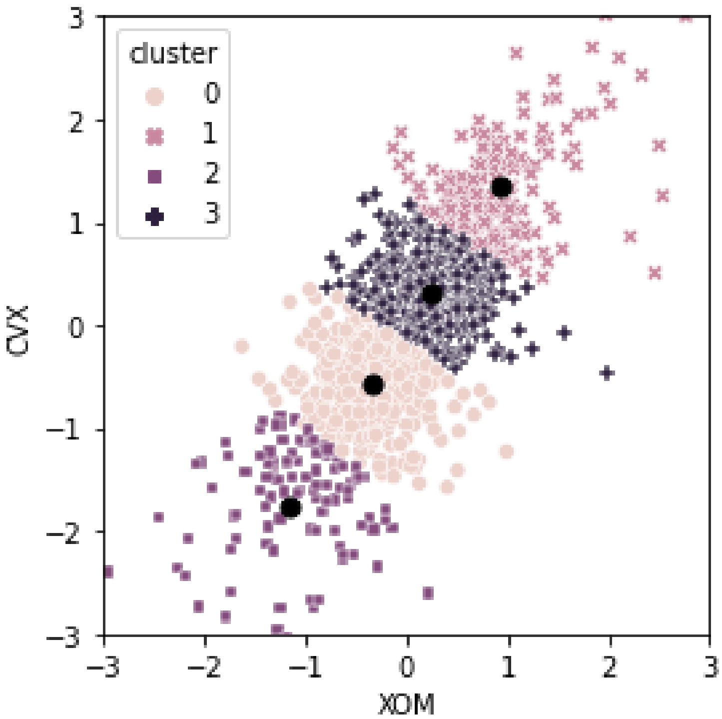
In [12]:

```
centers = pd.DataFrame(kmeans.cluster_centers_, columns=['XOM', 'CVX'])
print(centers)
```

```
        XOM       CVX
0 -0.329592 -0.567893
1  0.932608  1.354480
2 -1.143980 -1.750297
3  0.238172  0.321679
```

In [13]:

```
fig, ax = plt.subplots(figsize=(4, 4))
ax = sns.scatterplot(x='XOM', y='CVX', hue='cluster', style='cluster',
                     ax=ax, data=df)
ax.set_xlim(-3, 3)
ax.set_ylim(-3, 3)
centers.plot.scatter(x='XOM', y='CVX', ax=ax, s=50, color='black')

plt.tight_layout()
plt.show()
```



K-Means Algorithm

The *scikit-learn* algorithm is repeated 10 times by default (n_init), max_iter is used to control the number of iterations.

In [14]:

```
syms = sorted(['AAPL', 'MSFT', 'CSCO', 'INTC', 'CVX', 'XOM', 'SLB', 'COP',
               'JPM', 'WFC', 'USB', 'AXP', 'WMT', 'TGT', 'HD', 'COST'])
top_sp = sp500_px.loc[sp500_px.index >= '2011-01-01', syms]
kmeans = KMeans(n_clusters=5).fit(top_sp)
```

Interpreting the Clusters

In [15]:

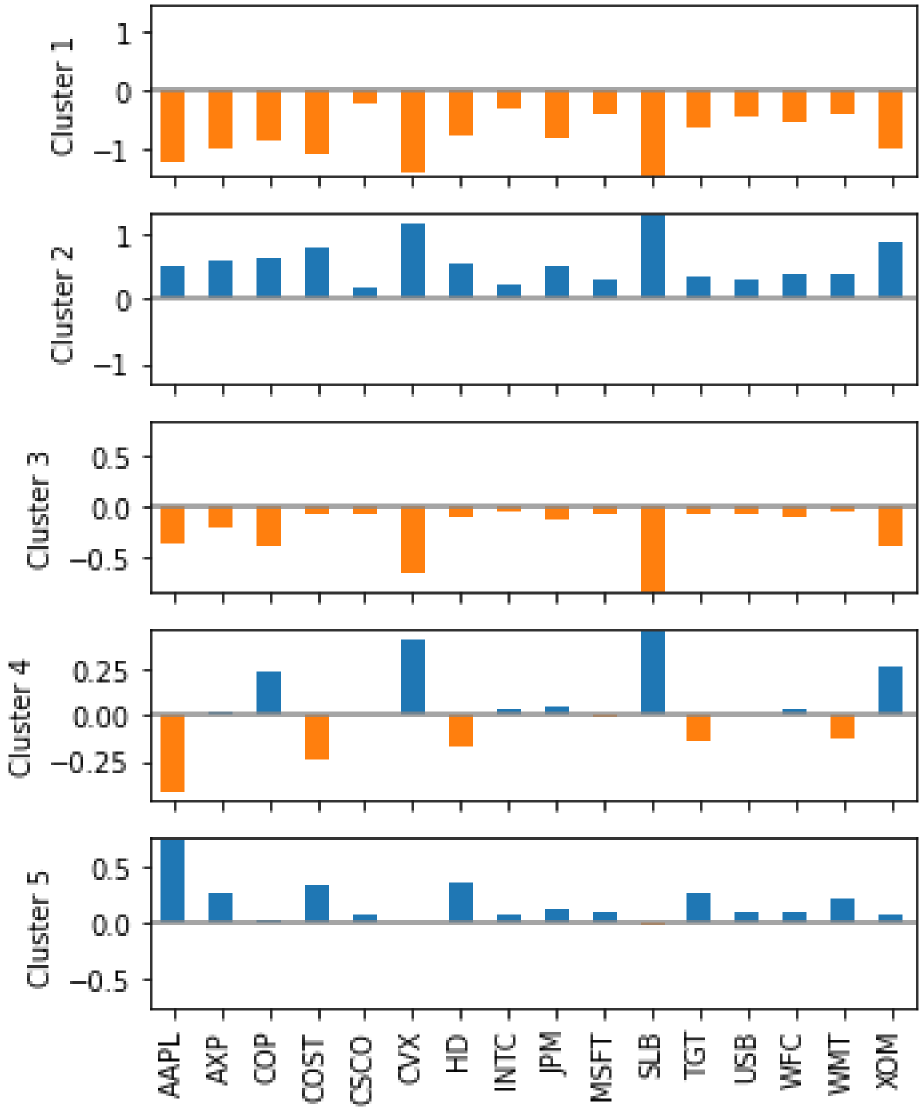
```
from collections import Counter
```

```
print(Counter(kmeans.labels_))
```

```
Counter({3: 290, 4: 284, 2: 272, 1: 174, 0: 111})
```

In [16]:

```
centers = pd.DataFrame(kmeans.cluster_centers_, columns=syms)

f, axes = plt.subplots(5, 1, figsize=(5, 6), sharex=True)
for i, ax in enumerate(axes):
    center = centers.loc[i, :]
    maxPC = 1.01 * np.max(np.max(np.abs(center)))
    colors = ['C0' if l > 0 else 'C1' for l in center]
    ax.axhline(color='#888888')
    center.plot.bar(ax=ax, color=colors)
    ax.set_ylabel(f'Cluster {i + 1}')
    ax.set_ylim(-maxPC, maxPC)

plt.tight_layout()
plt.show()
```

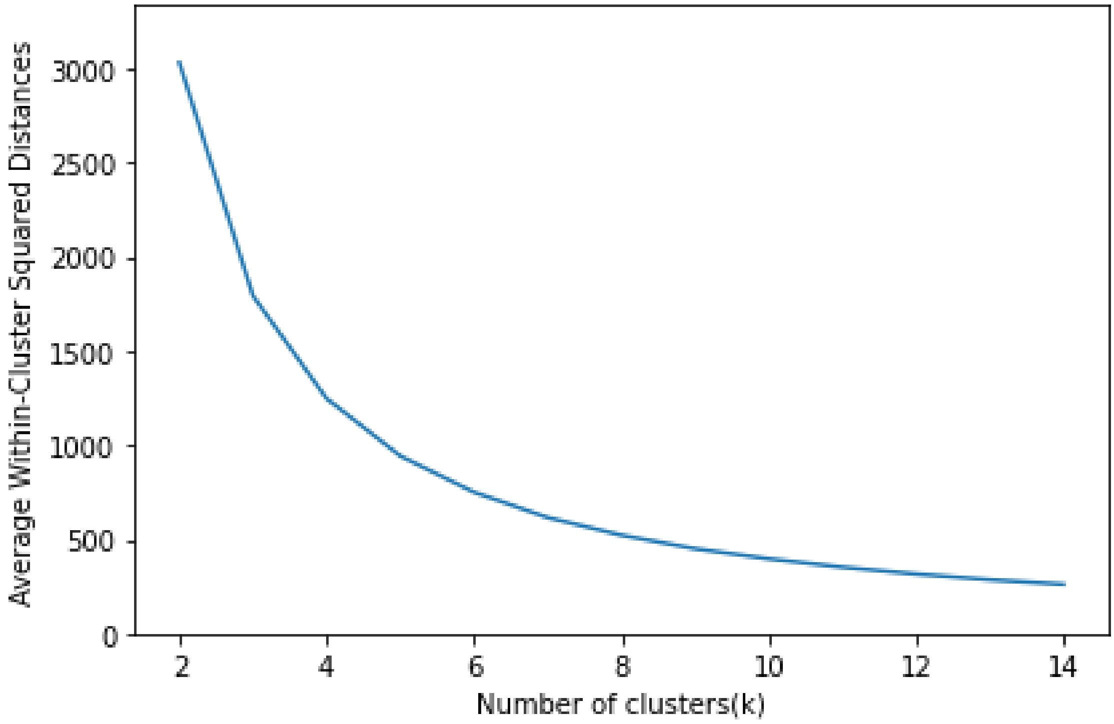


Selecting the Number of Clusters

In [17]:

```
inertia = []
```

```
for n_clusters in range(2, 15):
    kmeans = KMeans(n_clusters=n_clusters, random_state=0).fit(top_sp)
    inertia.append(kmeans.inertia_ / n_clusters)
inertias = pd.DataFrame({'n_clusters': range(2, 15), 'inertia': inertia})
ax = inertias.plot(x='n_clusters', y='inertia')
plt.xlabel('Number of clusters(k)')
plt.ylabel('Average Within-Cluster Squared Distances')
plt.ylim((0, 1.1 * inertias.inertia.max()))
ax.legend().set_visible(False)

plt.tight_layout()
plt.show()
```



Hierarchical Clustering

A Simple Example

In [18]:

```
syms1 = ['AAPL', 'AMZN', 'AXP', 'COP', 'COST', 'CSCO', 'CVX', 'GOOGL', 'HD',
         'INTC', 'JPM', 'MSFT', 'SLB', 'TGT', 'USB', 'WFC', 'WMT', 'XOM']
df = sp500_px.loc[sp500_px.index >= '2011-01-01', syms1].transpose()

Z = linkage(df, method='complete')
print(Z.shape)
```

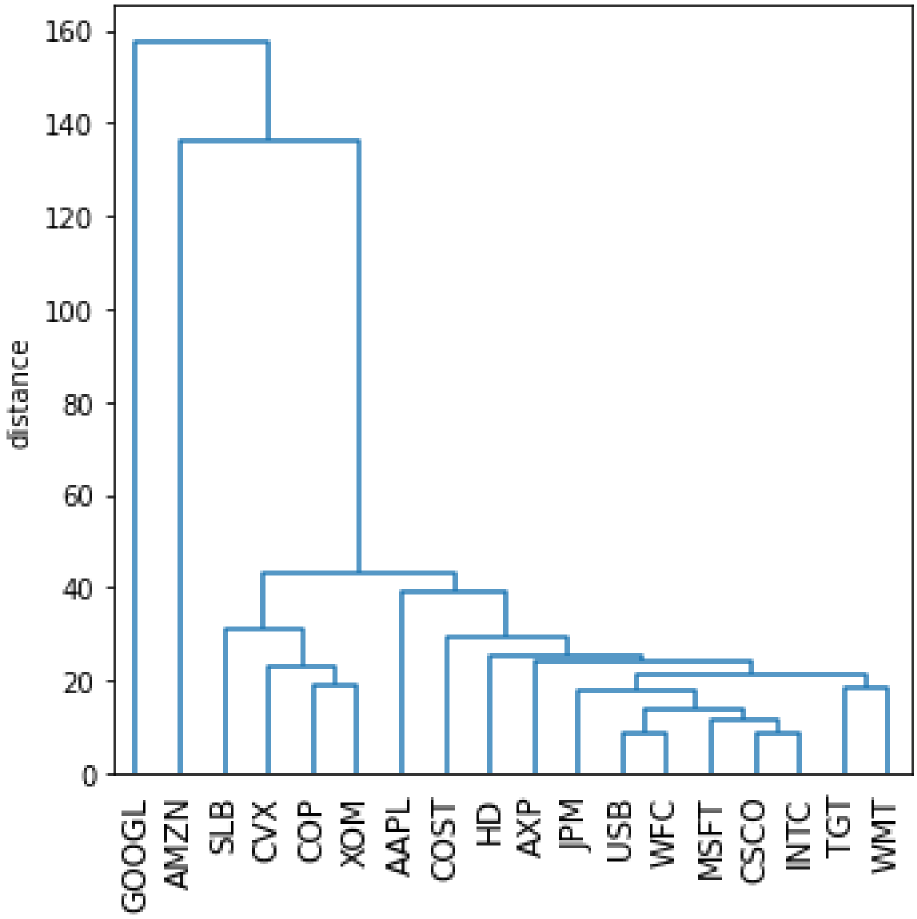
```
(17, 4)
```

The Dendrogram

In [19]:

```
fig, ax = plt.subplots(figsize=(5, 5))
dendrogram(Z, labels=list(df.index), color_threshold=0)
plt.xticks(rotation=90)
```

```
ax.set_ylabel('distance')

plt.tight_layout()
plt.show()
```



In [20]:

```
memb = fcluster(Z, 4, criterion='maxclust')
memb = pd.Series(memb, index=df.index)
for key, item in memb.groupby(memb):
    print(f"{key} : {', '.join(item.index)}")
```

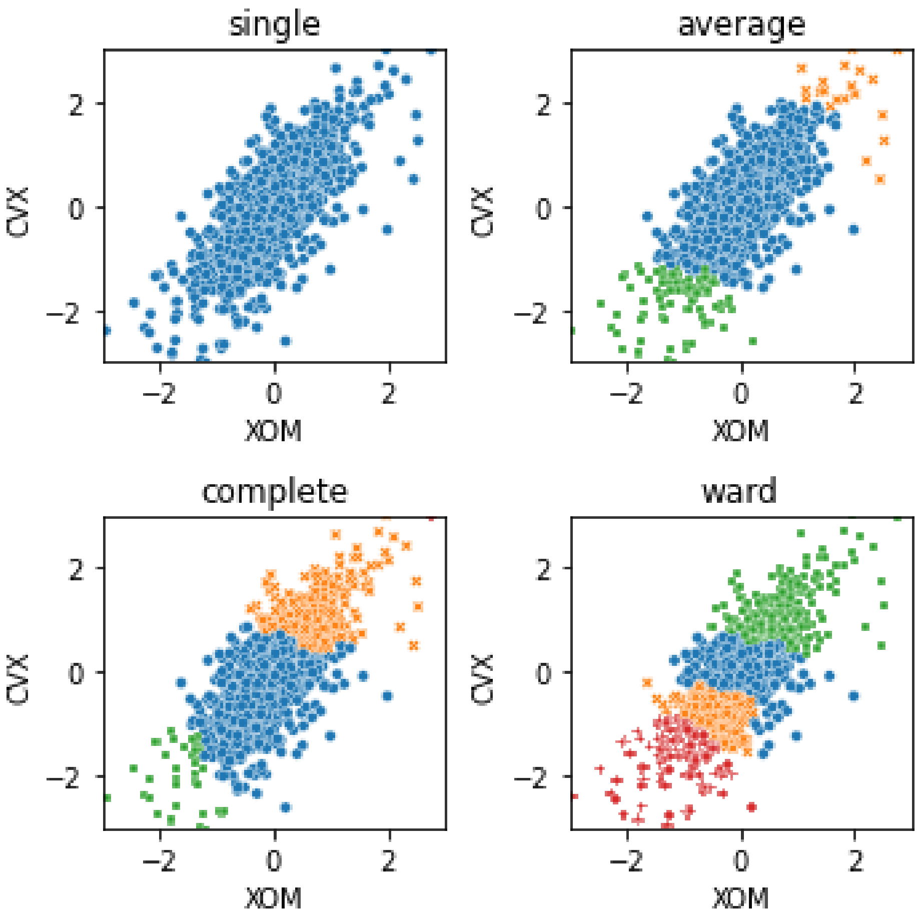
```
1 : COP, CVX, SLB, XOM
2 : AAPL, AXP, COST, CSCO, HD, INTC, JPM, MSFT, TGT, USB, WFC, WMT
3 : AMZN
4 : GOOGL
```

Measures of Dissimilarity

In [21]:

```
df = sp500_px.loc[sp500_px.index >= '2011-01-01', ['XOM', 'CVX']]
fig, axes = plt.subplots(nrows=2, ncols=2, figsize=(5, 5))
for i, method in enumerate(['single', 'average', 'complete', 'ward']):
    ax = axes[i // 2, i % 2]
    Z = linkage(df, method=method)
    colors = [f'C{c+1}' for c in fcluster(Z, 4, criterion='maxclust')]
    ax = sns.scatterplot(x='XOM', y='CVX', hue=colors, style=colors,
                         size=0.5, ax=ax, data=df, legend=False)

    ax.set_xlim(-3, 3)
    ax.set_ylim(-3, 3)
```

```
    ax.set_title(method)

plt.tight_layout()
plt.show()
```



Model based clustering

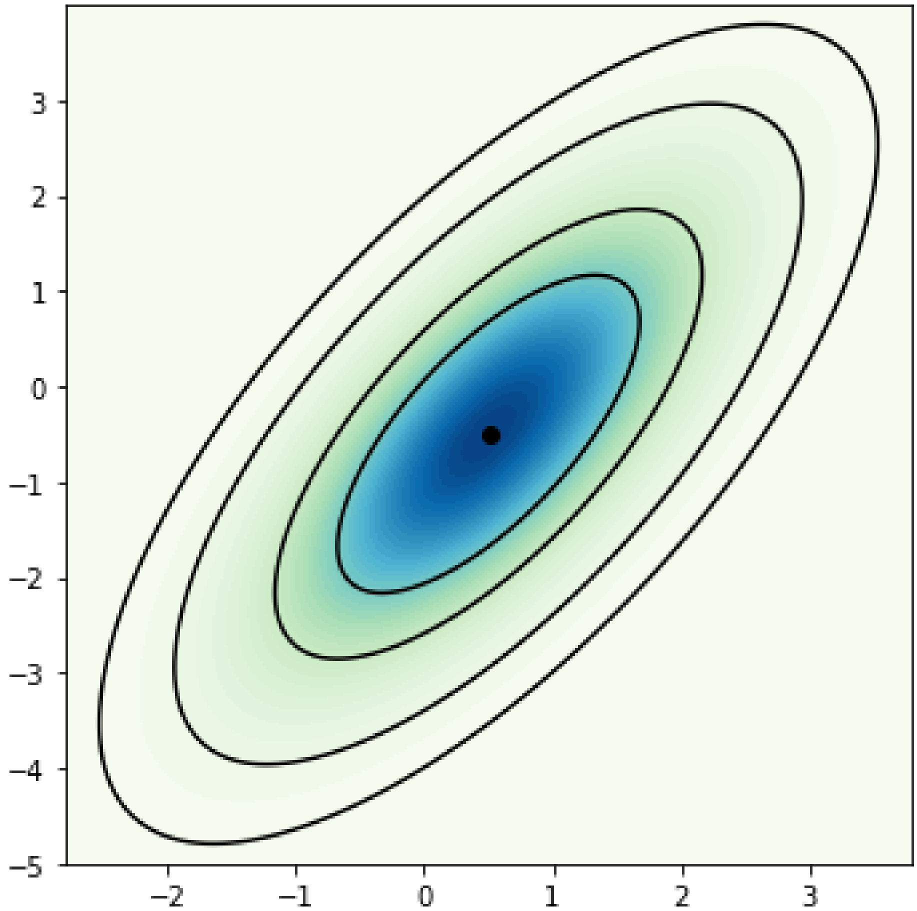
Multivariate Normal Distribution

Define a colormap that corresponds to the probability levels

In [22]:

```
mean = [0.5, -0.5]
cov = [[1, 1], [1, 2]]
probability = [.5, .75, .95, .99]
def probLevel(p):
    D = 1
    return (1 - p) / (2 * math.pi * D)
levels = [probLevel(p) for p in probability]

fig, ax = plt.subplots(figsize=(5, 5))

x, y = np.mgrid[-2.8:3.8:.01, -5:4:.01]
pos = np.empty(x.shape + (2,))
pos[:, :, 0] = x; pos[:, :, 1] = y
rv = multivariate_normal(mean, cov)
```

```
CS = ax.contourf(x, y, rv.pdf(pos), cmap=cm.GnBu, levels=50)
ax.contour(CS, levels=levels, colors=['black'])
ax.plot(*mean, color='black', marker='o')


plt.tight_layout()
plt.show()
```



Mixtures of Normals

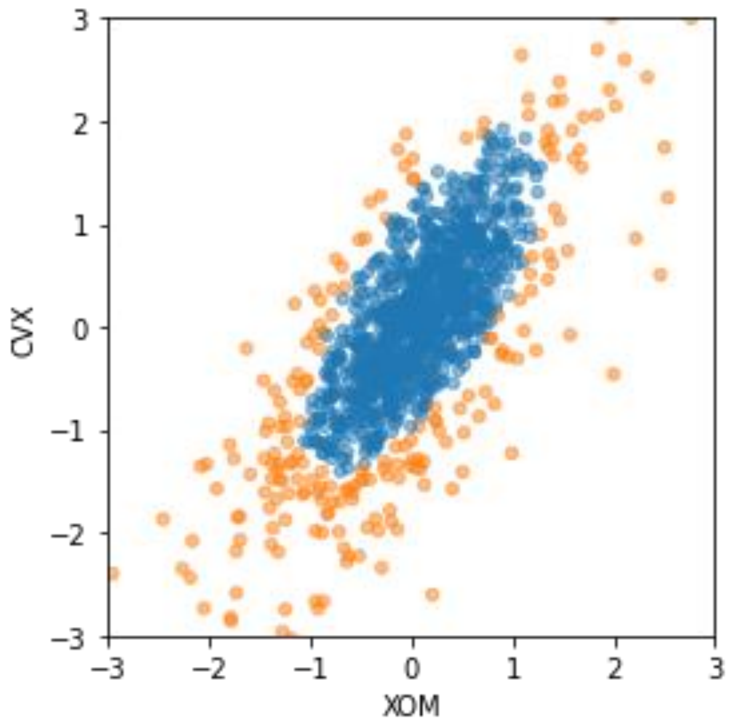
In [23]:

```
df = sp500_px.loc[sp500_px.index >= '2011-01-01', ['XOM', 'CVX']]
mclust = GaussianMixture(n_components=2).fit(df)
print(mclust.bic(df))
```

```
4589.820626249872
```

In [24]:

```
fig, ax = plt.subplots(figsize=(4, 4))
colors = [f'C{c}' for c in mclust.predict(df)]
df.plot.scatter(x='XOM', y='CVX', c=colors, alpha=0.5, ax=ax)
ax.set_xlim(-3, 3)
ax.set_ylim(-3, 3)

plt.tight_layout()
plt.show()
```



In [25]:

```
print('Mean')
print(mclust.means_)
print('Covariances')
print(mclust.covariances_)
```

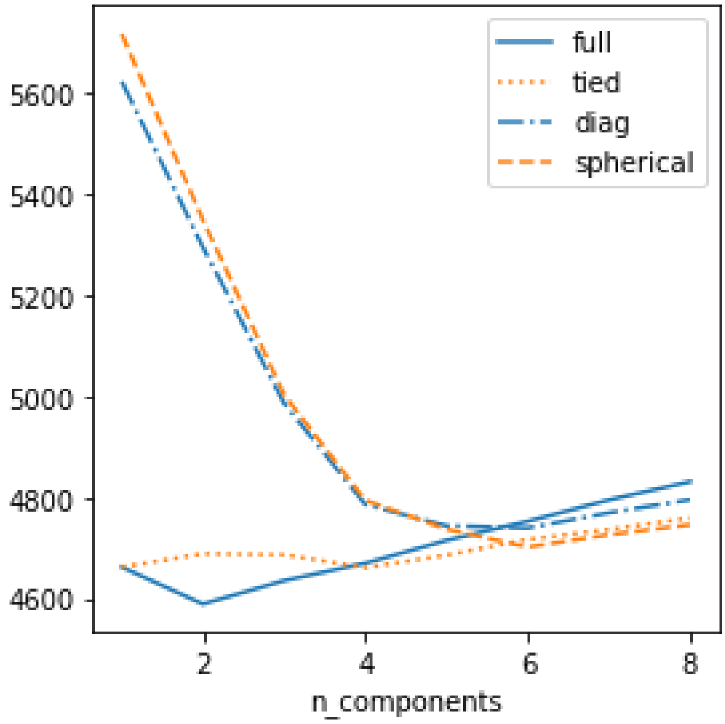
```
Mean
[[ 0.07225117  0.10452744]
 [-0.05050178 -0.21237957]]
Covariances
[[[0.26868436 0.27606914]
  [0.27606914 0.51762673]]

 [[0.97385279 0.98028909]
  [0.98028909 1.67646834]]]
```

Selecting the number of clusters

In [26]:

```
results = []
covariance_types = ['full', 'tied', 'diag', 'spherical']
for n_components in range(1, 9):
    for covariance_type in covariance_types:
        mclust = GaussianMixture(n_components = n_components,
warm_start=True,
                                 covariance_type = covariance_type)
        mclust.fit(df)
        results.append({
            'bic': mclust.bic(df),
            'n_components': n_components,
            'covariance_type': covariance_type,
        })

results = pd.DataFrame(results)

colors = ['C0', 'C1', 'C2', 'C3']
```

```
styles = ['C0-','C1:','C0-.', 'C1--']

fig, ax = plt.subplots(figsize=(4, 4))
for i, covariance_type in enumerate(covariance_types):
    subset = results.loc[results.covariance_type == covariance_type, :]
    subset.plot(x='n_components', y='bic', ax=ax, label=covariance_type,
                kind='line', style=styles[i]) # , color=colors[i])

plt.tight_layout()
plt.show()
```



Scaling and Categorical Variables

Scaling the Variables

In [27]:

```
loan_data = pd.read_csv(LOAN_DATA_CSV)
loan_data['outcome'] = pd.Categorical(loan_data['outcome'],
                                      categories=['paid off', 'default'],
                                      ordered=True)
defaults = loan_data.loc[loan_data['outcome'] == 'default',]

columns = ['loan_amnt', 'annual_inc', 'revol_bal', 'open_acc',
           'dti', 'revol_util']

df = defaults[columns]
kmeans = KMeans(n_clusters=4, random_state=1).fit(df)
counts = Counter(kmeans.labels_)

centers = pd.DataFrame(kmeans.cluster_centers_, columns=columns)
centers['size'] = [counts[i] for i in range(4)]
print(centers)
```

```
      loan_amnt      annual_inc      revol_bal    open_acc          dti   \
```

```
0  18275.132345   83354.634595  19635.189254  11.664373  16.774586
1  21852.701005  165407.730318  38907.295645  12.597152  13.466876
2  10591.893792   42453.058692  10268.048598   9.583820  17.713563
3  22570.192308  489783.403846  85161.346154  13.326923   6.907500

   revol_util   size
0   62.258588   7543
1   63.634900   1194
2   58.111226  13882
3   59.651923     52
```

In [28]:

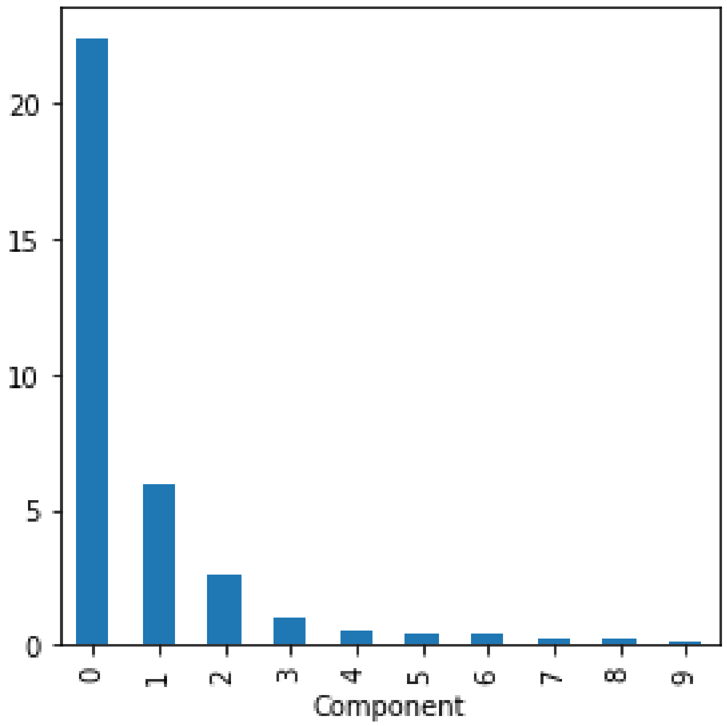
```
scaler = preprocessing.StandardScaler()
df0 = scaler.fit_transform(df * 1.0)

kmeans = KMeans(n_clusters=4, random_state=1).fit(df0)
counts = Counter(kmeans.labels_)

centers = pd.DataFrame(scaler.inverse_transform(kmeans.cluster_centers_),
                       columns=columns)
centers['size'] = [counts[i] for i in range(4)]
print(centers)
```

```
      loan_amnt     annual_inc     revol_bal   open_acc        dti  \
0  10499.824632   51070.958451  11629.172535   7.511129  15.965747
1  10315.255666   53468.181307   6032.616033   8.637385  11.255855
2  25920.260952  116308.326663  32827.641428  12.389941  16.204021
3  13420.700048   55844.852918  16370.832021  14.334512  24.189881

   revol_util  size
0   77.806693  7405
1   31.000342  5339
2   66.172004  3701
3   59.227862  6226
```

Dominant Variables

In [29]:

```
syms = ['GOOGL', 'AMZN', 'AAPL', 'MSFT', 'CSCO', 'INTC', 'CVX', 'XOM',
        'SLB', 'COP', 'JPM', 'WFC', 'USB', 'AXP', 'WMT', 'TGT', 'HD', 'COST']
top_sp1 = sp500_px.loc[sp500_px.index >= '2005-01-01', syms]

sp_pca1 = PCA()
sp_pca1.fit(top_sp1)

explained_variance = pd.DataFrame(sp_pca1.explained_variance_)
ax = explained_variance.head(10).plot.bar(legend=False, figsize=(4, 4))
ax.set_xlabel('Component')

plt.tight_layout()
plt.show()
```



In [30]:

```
loadings = pd.DataFrame(sp_pca1.components_[0:2, :],
                        columns=top_sp1.columns)
print(loadings.transpose())
```

```
              0         1
GOOGL -0.857310  0.477873
AMZN  -0.444728 -0.874149
AAPL  -0.071627 -0.020802
MSFT  -0.036002 -0.006204
CSCO  -0.029205 -0.003045
INTC  -0.026666 -0.006069
CVX   -0.089548 -0.037420
XOM   -0.080336 -0.020511
SLB   -0.110218 -0.030356
COP   -0.057739 -0.024117
JPM   -0.071228 -0.009244
WFC   -0.053228 -0.008597
USB   -0.041670 -0.005952
AXP   -0.078907 -0.024027
WMT   -0.040346 -0.007141
TGT   -0.063659 -0.024662
HD    -0.051412 -0.032922
COST  -0.071403 -0.033826
```

Categorical Data and Gower's Distance

Currently not available in any of the standard packages. However work is in progress to add it to scikit-learn. We will update this notebook once it becomes available

https://github.com/scikit-learn/scikit-learn/pull/9555/

In [31]:

```
x = defaults[['dti', 'payment_inc_ratio', 'home_', 'purpose_']].loc[0:4, :]
print(x)
```

```
     dti  payment_inc_ratio home_            purpose_
```

```
0   1.00             2.39320  RENT      major_purchase
1   5.55             4.57170   OWN      small_business
2  18.08             9.71600  RENT               other
3  10.08            12.21520  RENT  debt_consolidation
4   7.06             3.90888  RENT               other
```

```
################################################################

## Figure 7-13: Categorical data and Gower's distance



x <- loan_data[1:5, c('dti', 'payment_inc_ratio', 'home_', 'purpose_')]

x



daisy(x, metric='gower')



set.seed(301)

df <- loan_data[sample(nrow(loan_data), 250),

                c('dti', 'payment_inc_ratio', 'home_', 'purpose_')]

d = daisy(df, metric='gower')

hcl <- hclust(d)

dnd <- as.dendrogram(hcl)



png(filename=file.path(PSDS_PATH, 'figures', 'psds_0713.png'), width = 4, height=4, units='in', res=300)

par(mar=c(0,5,0,0)+.1)

plot(dnd, leaflab='none', ylab='distance')

dev.off()



dnd_cut <- cut(dnd, h=.5)
```

df[labels(dnd_cut$lower[[1]]),]

Problems in clustering with mixed data types

df <- model.matrix(~ -1 + dti + payment_inc_ratio + home_ + pub_rec_zero, data=defaults)

df0 <- scale(df)

km0 <- kmeans(df0, centers=4, nstart=10)

centers0 <- scale(km0$centers, center=FALSE, scale=1/attr(df0, 'scaled:scale'))

round(scale(centers0, center=-attr(df0, 'scaled:center'), scale=FALSE), 2)

Problems with Clustering Mixed Data

In [32]:

```
columns = ['dti', 'payment_inc_ratio', 'home_', 'pub_rec_zero']
df = pd.get_dummies(defaults[columns])

scaler = preprocessing.StandardScaler()

df0 = scaler.fit_transform(df * 1.0)
kmeans = KMeans(n_clusters=4, random_state=1).fit(df0)
centers = pd.DataFrame(scaler.inverse_transform(kmeans.cluster_centers_),
                       columns=df.columns)
print(centers)
```

```
         dti  payment_inc_ratio  pub_rec_zero  home__MORTGAGE     home__OWN  \
0  16.992128           9.105395      1.000000    1.171285e-14 -1.346145e-15
1  17.456244           8.422914      1.000000    1.000000e+00 -1.193490e-15
2  16.504955           8.064247      0.000000    5.156600e-01  1.110223e-16
3  17.197993           9.266666      0.917903   -6.106227e-16  1.000000e+00

     home__RENT
0  1.000000e+00
1 -2.164935e-15
2  4.843400e-01
3  1.998401e-15
```